**Lab 11  
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1. Starting with the values 1, 2, 4, 4, 5, 6, 9, 11, 12, 12, 17, do the following:
2. Create a heap H in which these values are the keys.

**A picture containing map, sky

Description generated with high confidence**

1. Perform the insertItem algorithm to insert the value 7 into H. Show all steps.

A picture containing map, sky

Description generated with high confidence

1. Perform the removeMin algorithm on H and show all steps.

**A picture containing map, sky

Description generated with high confidenceA picture containing map, sky

Description generated with very high confidenceA close up of a map

Description generated with high confidence**

1. Represent H in the form of an array A.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **2** | **4** | **4** | **7** | **5** | **6** | **9** | **11** | **12** | **12** | **17** |  |

1. Perform the array-based insertItem algorithm to insert 14 into A – show all steps.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **2** | **4** | **4** | **7** | **5** | **6** | **9** | **11** | **12** | **12** | **17** | **14** |

1. Perform the array-based removeMin algorithm on A – show all steps.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **14** | **4** | **4** | **7** | **5** | **6** | **9** | **11** | **12** | **12** | **17** |  |

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|  | **4** | **14** | **4** | **7** | **5** | **6** | **9** | **11** | **12** | **12** | **17** |  |

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|  | **4** | **5** | **4** | **7** | **14** | **6** | **9** | **11** | **12** | **12** | **17** |  |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **4** | **5** | **4** | **7** | **12** | **6** | **9** | **11** | **12** | **14** | **17** |  |

1. Carry out the array-based version of HeapSort on the input array

[1, 4, 3, 9, 12, 2, 4]

Show steps and outputs along the way. Make sure to distinguish between Phase I and Phase II of the algorithm.

**Phase I:**

|  |  |
| --- | --- |
| **1** | **4 3 9 12 2 4** |
| **4 1** | **3 9 12 2 4** |
| **3 4 1** | **3 9 12 2 4** |
| **4 3 1** | **9 12 2 4** |
| **9 4 3 1** | **12 2 4** |
| **12 9 4 3 1** | **2 4** |
| **2 12 9 4 3 1** | **4** |
| **12 2 9 4 3 1** | **4** |
| **12 4 9 2 3 1** | **4** |
| **4 12 4 9 2 3 1** |  |
| **12 4 4 9 2 3 1** |  |
| **12 9 4 4 2 3 1** |  |

**Phase II:**

|  |  |
| --- | --- |
| **9 4 4 2 3 1** | **12** |
| **4 4 2 3 1** | **9 12** |
| **4 2 3 1** | **4 9 12** |
| **2 3 1** | **4 4 9 12** |
| **3 2 1** | **4 4 9 12** |
| **2 1** | **3 4 4 9 12** |
| **1** | **2 3 4 4 9 12** |
|  | **1 2 3 4 4 9 12** |

1. The recursive version of BottomUpHeap relies on the Proposition given below. For this exercise, prove the Proposition.

**Proposition.** Suppose n is a positive integer of the form 2h - 1 for some h. Then n may be written in the form n = 1 + m + m for some m. Moreover, m must equal 2h - 1 – 1.

**n = 2h – 1**

**= 2 . 2h – 1 – 1**

**= 2 ( 2h – 1 – 1) + 1**

**= m + m + 1**

**where m = 2h – 1 - 1**

1. Carry out the steps of the recursive algorithm BottomUpHeap for the input sequence 11, 5, 2, 3, 17, 24, 1

**BUH([11, 5, 2, 3, 17, 24, 1])**

**k = 11, A1 = {5, 2, 3}, A2 = {17, 24, 1}**

**A1: k = 5, A1 = {2}, A2 = {3}**

**A close up of a necklace

Description generated with very high confidence**

**A2: k = 17, A1 = {24}, A2 = {1}**

**A drawing of a person

Description generated with high confidence A drawing of a person

Description generated with very high confidence**

**k = 11, A1 + A2**

**A picture containing skiing, sky

Description generated with very high confidence**

**A picture containing skiing, sky

Description generated with high confidence**

**A picture containing skiing, sky

Description generated with high confidence**

1. Prove the following facts about numbers:
2. 3 | x2 – 1 for any x that is not a multiple of 3
3. 5 | x4 – 1 for any x that is not a multiple of 5

Rewrite these statements using mod notation. Then make a guess about the general result: What general fact are these two problems special cases of?

1. **x is not a multiple of 3 if for some k and h where 0 < k < 3, such that x = 3h + k**

**Suppose x = 3h + 1**

**X2 – 1 = (3h + 1)2 – 1**

**= 9h2 + 6h (divisible by 3)**

**Suppose x = 3h + 2**

**X2 – 1 = (3h + 2)2 – 1**

**= 9h2 + 12h + 3 (divisible by 3)**

1. **x is not a multiple of 5 if for some k and h where 0 < k < 5, such that x = 5h + k**

**From (i) above, I assume the same holds for (ii)**

**In General: P | xP – 1 for any x that is not a multiple of P**

**General fact of modular notation: xP – 1 ≡ 1 mod P**

1. Let *Z3* = {0,1,2} and *Z5* = {0,1,2,3,4}. Which numbers in *Z3* are perfect squares? That is, for which numbers a Ɛ *Z3* does there exist an x Ɛ *Z3* such that x2 **≡** a mod 3? Which numbers *a in Z5* are perfect squares? These numbers a are called quadratic residues mod 3 (or 5). For any odd prime p, how many quadratic residues are there mod p?

**In Z3 the set {1, 2} exists such that x2 ≡ a mod 3**

**In Z5 the set {4} consists of perfect squares**

**For any odd prime P, there exists P – 1 quadratic residues.**